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Compatible Growth and Yield Equations For West Virginia Yellow-Poplar

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Compatible Growth and Yield Equations For West Virginia Yellow-Poplar

BRYCE E. SCHLAEGEL and D. L. KULOW

Yellow-poplar (**Liriodendron tulipifera L.**) is the dominant species in its second growth "yellow-poplar type" forest and in the yellow-poplar — white oak — red oak type forest. These two forest types occupy approximately one-fifth of the commercial forest area of West Virginia, and contain 20 per cent of the saw timber volume in the State (7).[1] Accurate prediction of growth and yield in these stands in an integral part of effective timber management.

Growth and yield are related variables. The summation of the predicted growth values to a particular age should produce the yield at that age. This has not been true previously because of the difficulty of finding the mathematical relationship between these two variables. The purpose of this bulletin was to find a suitable model for growth that would give compatible yield results. The growth models were fitted to the data, and tested to see if such a mathematical relationship does exist.

REVIEW OF LITERATURE

The first yield tables for even-aged stands in the United States were constructed in 1894 for white pine by Gifford Pinchot and Henry S. Graves (6). These tables were made by fitting a freehand curve to data for fully stocked or "normal" stands. Bruce (1) recognized the problems involved in fitting series of freehand curves and suggested the use of anamorphasis.[2] In 1926, he constructed straight-line yield table curves of volume over age by site classes (2). He found the curves to be of the same shape for all site index classes. Reineke (10) modified Bruce's method by using alignment charts instead of the anamorphasis curves.

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[1] Numbers in parentheses refer to literature cited.

[2] Anamorphasis is a freehand method used to convert curvilinear graphs to straight line functions.

Dwight (5) suggested the use of growth charts in place of the yield table method to predict the yield of even-aged stands. He used present diameter as a basis to find future diameter and volume of each individual tree. In the yield table method, future yield is based upon the present cubic or board-foot volume of the stand. He recognized, however, that the errors in the growth chart method may be large, since it is assumed that trees of a certain size at a given age will continue to grow at approximately the same rates, even in stands of different densities.

Bruce and Reineke (3) were among the first to combine graphics and statistics and produced alignment charts suitable for yield tables. The graphics allowed flexibility, while the statistics provided mathematical accuracy. The process of Bruce and Reineke involved calculating a multiple regression equation and then converting that equation into an alignment chart. This method presupposes that the coefficient of variation of yield for the stand is the same at all ages. However, Osborne and Schumacher (9) showed for "normal" stands of red gum that the coefficients of variation for heights of dominant trees in the stand, the number of trees, and the volume are dependent upon the age of the stand. They demonstrated a method of constructing yield tables that were dependent upon the coefficient of variation at each age class. They found that the relation of the coefficient of variation of yield to stand age can be used to determine the form of the growth curve of any site index from the growth curve of the average site index.

Until 1937 it was felt that the "normality" concept was the ultimate in forest management. A few foresters realized that the attainment of a "normal" forest was impractical, if not impossible. MacKinney, Schumacher, and Chaiken (8) used correlation analysis to find the mathematical relationship between the dependent variable volume and the independent variables of age, site, and stand density. They produced the first yield tables for "non-normal" loblolly pine stands. However, the summation of the growth values failed to produce the yield value given in the table for a predicted age. In 1963, Clutter (4) developed "compatible" growth and yield equations. His work was based on the mathematical dependency that must exist between growth and yield. With such equations, it is possible to predict yield (a relatively difficult variable to measure) from growth (a relatively easy variable to measure). His procedure is used in this study on the first measurements of yellow-poplar growth and yield.

METHODS AND PROCEDURES

The data used in this study was collected in 1964 and 1965 from 123 permanent sample plots located in Greenbrier, Monongalia, Preston, Randolph, and Nicholas counties, West Virginia. The plots ranged in size from one-tenth to one-quarter acre, with an isolation strip of one-half chain. The composition of the overstory consisted of at least 75 per cent yellow-poplar that varied in age from 26 to 80 years and on sites of 57 to 110 feet.

All yellow-poplar over one-half inch and all other species over 4.5 inches were tallied by species and diameter. A sample of these trees on each plot was measured for dbh, height to an eight-inch top, height to a four-inch top, total height, crown class, age, bark thickness, and width of the last ten years of growth. Plot volume was determined by the local volume table method.[3]

Since only the initial measurement has been made in this study, plot growth was determined via the stand table projection method, using the last ten years radial growth. All of the advantages and disadvantages of this procedure are assumed. A number of yield models were selected from the literature (Appendix A) and fitted to the data. All models showed a significant "fit" to the data, but only the one with the largest R-squared value was used in this study. This was the same one used by Clutter (1963). This yield model was differentiated with respect to age to give a growth model representing volume change at a moment in time. This model was then fitted to the results of the stand projection method and tested for "goodness of fit." This growth equation was then integrated into a prediction equation for yield. By summing this equation over periods of time, compatible yields were obtained. The steps in this procedure are shown in Appendix B.

RESULTS AND DISCUSSION

Using least squares techniques on model forms resulted in a set of five equations that can be used to establish growth and

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[3] The relationship between height and diameter for the sample trees was found by regression. Volumes for selected diameters and calculated heights were read from Schnur (1937). The relationship between volume in cubic feet inside bark for the entire stem and diameter was then calculated as:

$$\text{Volume} = 4.9227 - 1.6455D + 0.2596D^2$$

yield relationships present in data from the first measurements of yellow-poplar plots. These equations are:

Cubic-foot yield;

$$\mathrm{Ln}V = 4.7123 + 0.0071(S) + 0.6167(\mathrm{Ln}B) - 7.7335(A^{-1}) \tag{2}$$

Basal area growth rate;

$$dB/dA = B(5.9358 - \mathrm{Ln}B)A^{-1} \tag{8}$$

Cubic-foot growth rate;

$$dV/dA = V(1.5261 - 0.0380\ \mathrm{Ln}B)A^{-1} \tag{9}$$

Projected basal area;

$$\mathrm{Ln}B_P = 5.9358 - A_0A_P^{-1}(5.9358 - \mathrm{Ln}B_0) \tag{10}$$

Projected cubic-foot volume;

$$\mathrm{Ln}V_P = \mathrm{Ln}V_0 + 1.3003(\mathrm{Ln}\overset{A}{\blacksquare}_P - \mathrm{Ln}\overset{A}{\blacksquare}_0) + A_0(A_P^{-1} - A_0^{-1})\ (0.0380\ \mathrm{Ln}B_0 - 0.2258) \tag{12}$$

where:

$\mathrm{Ln}V$ = the natural logarithm of inside bark cubic-foot plot volume,

S = site index in feet,
B = stand basal area in square feet,
A = stand age in years,
dB/dA = instantaneous rate of basal area growth,
dV/dA = instantaneous rate of cubic-foot growth,
B_0 = initial basal area,
A_0 = initial age,
B_P = projected basal area,
A_P = predicted age,
V_P = projected cubic-foot yield per acre, and
V_0 = initial cubic-foot yield.

Graphs of the five equations have been plotted in Figures 1 to 6. Figures 1 and 2 show yields per acre for various age and basal area classes for site indices of 75 to 95 (Equation 2). These are essentially empirical yield tables, where the present stand yield can be solved for any combination of age, site, and basal area. Figure 3 shows the basal area growth rate (Equation 8) by age and basal area classes. It should also be noted that the basal area growth per acre increases with basal area until a basal area of 140 square feet is obtained. At that point the growth rate begins to decline. Figure 4 shows the annual cubic-foot growth rates that can be expected at various ages and basal areas for an initial volume of 3,400 cubic feet (Equation 9). Although any initial volume could be used in this equation, 3,400 cubic feet was used since it most nearly represents the present mean cubic-foot yield.

Residual values for both growth and yield were calculated and plotted over age. For all cases, the distribution of points were random enough to indicate no increase in variance with age.

Solutions of the basal area projection equation (Equation 10) appear in Figure 5. These curves show the expected basal area at various ages and initial basal areas by means of the growth rates shown in Figure 3. The curves for projected cubic-foot volume are shown in Figure 6. The values plotted are found by solving Equation 12 for an initial volume of 3,400 cubic feet at various ages and basal areas.

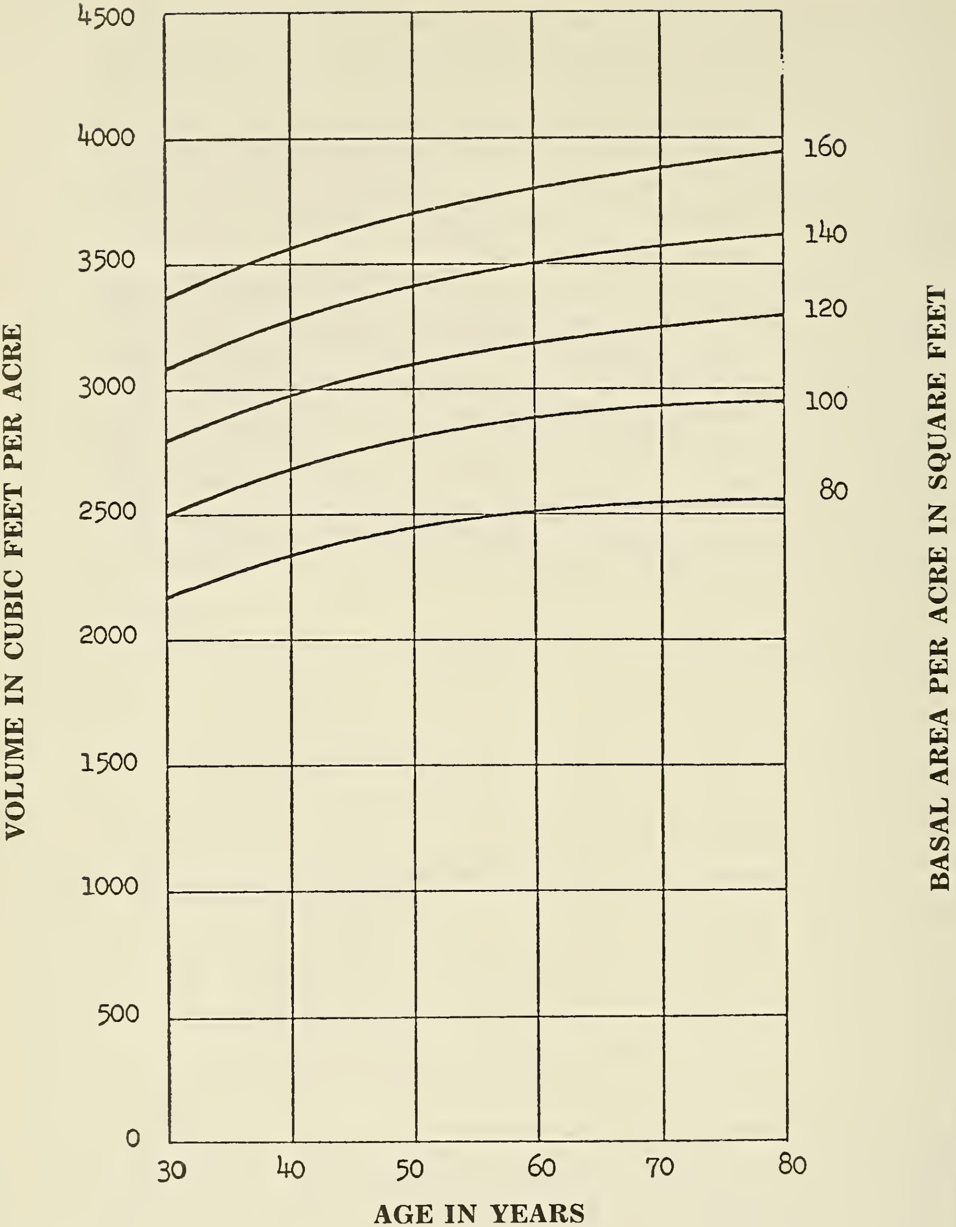


Fig. 1

Yellow-Poplar Yield in Cubic Feet per Acre Site Index 75

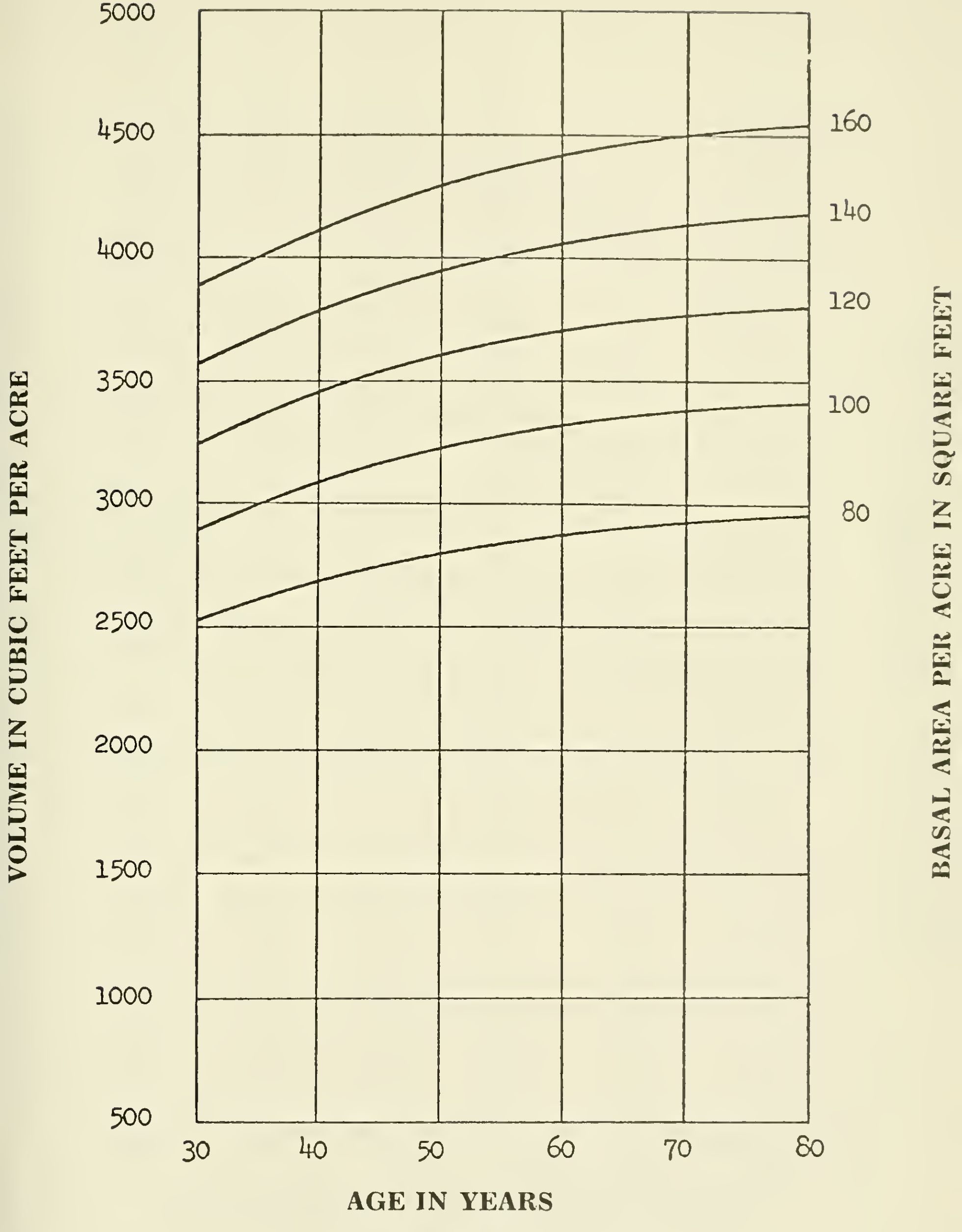
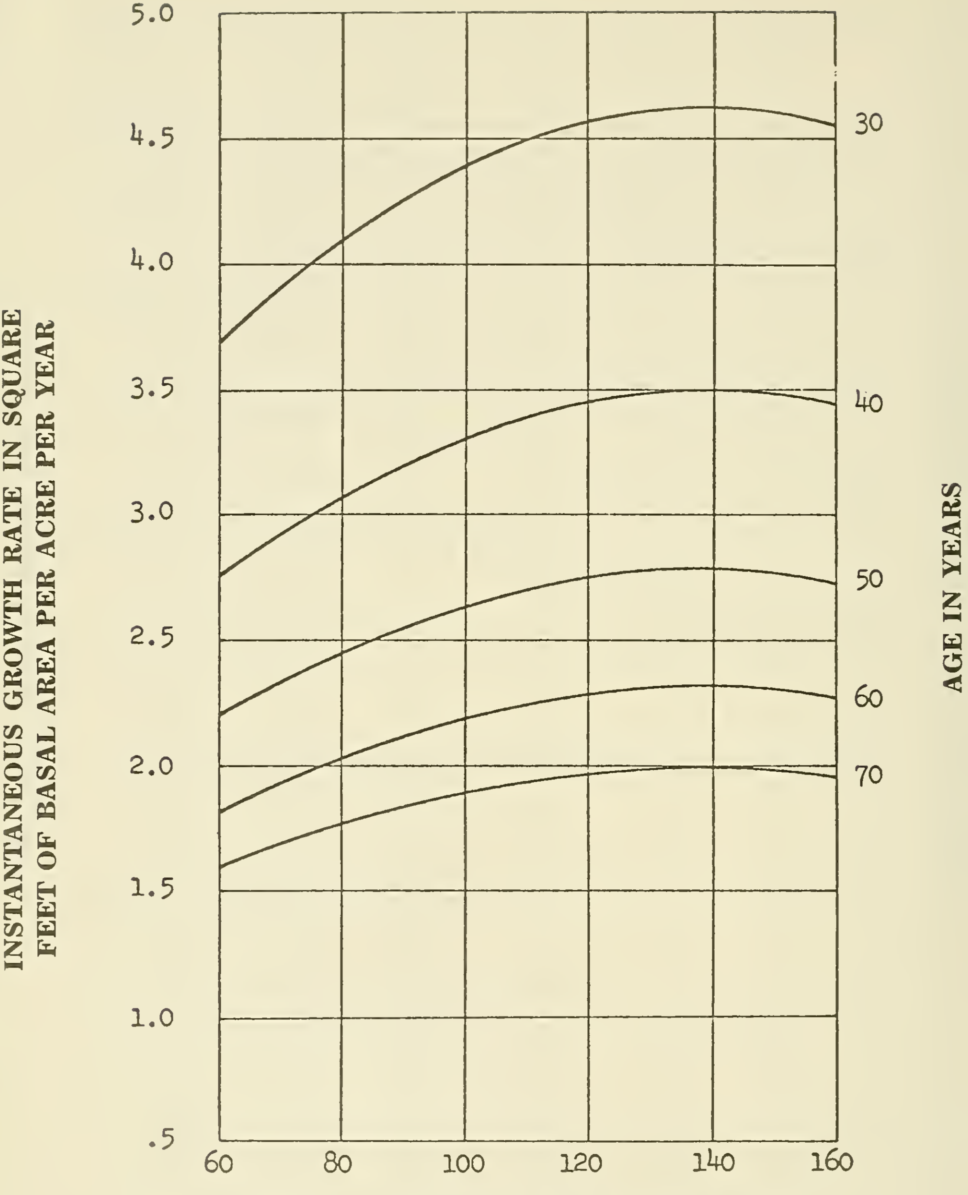


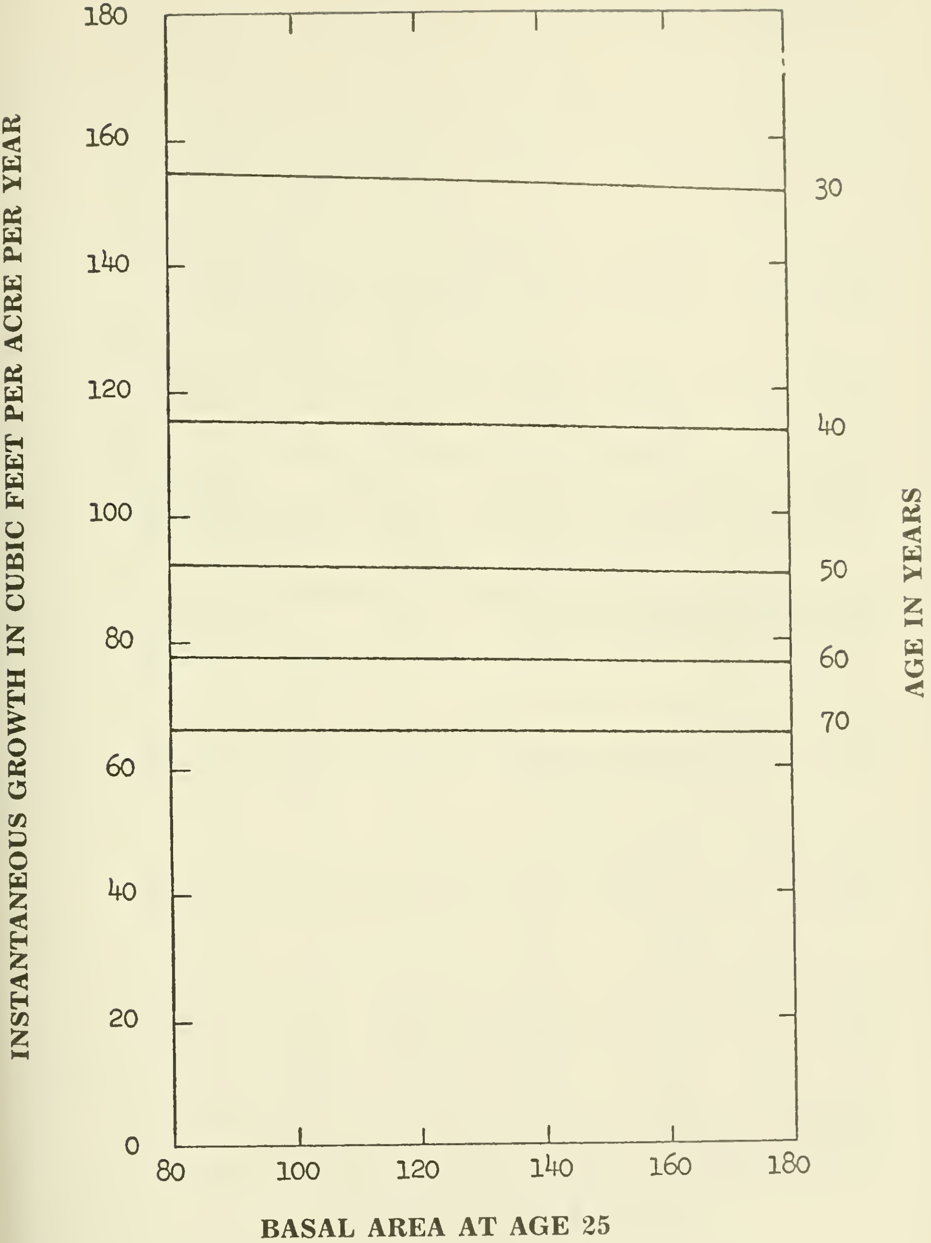
Fig. 2
Yellow-Poplar Yield in Cubic Feet
per Acre
Site Index 95



BASAL AREA PER ACRE IN SQUARE FEET

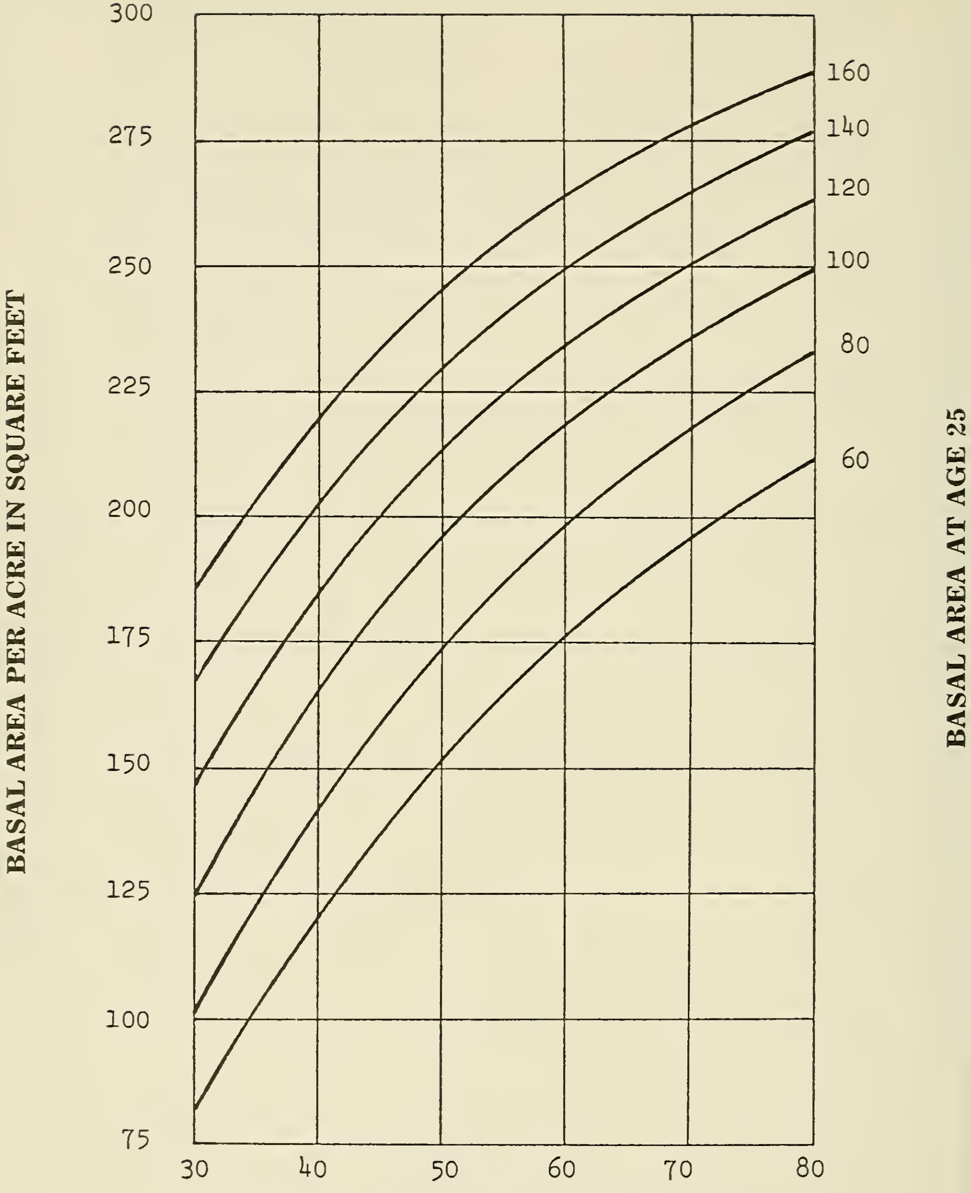
Fig. 3

Basal Area Growth in Square Feet per Acre per Year



BASAL AREA AT AGE 25

Fig. 4
Cubic-Foot Growth per Acre per Year,
Initial Volume 3400 Cubic Feet



AGE IN YEARS

Fig. 5
Projected Basal Area in Square Feet per Acre

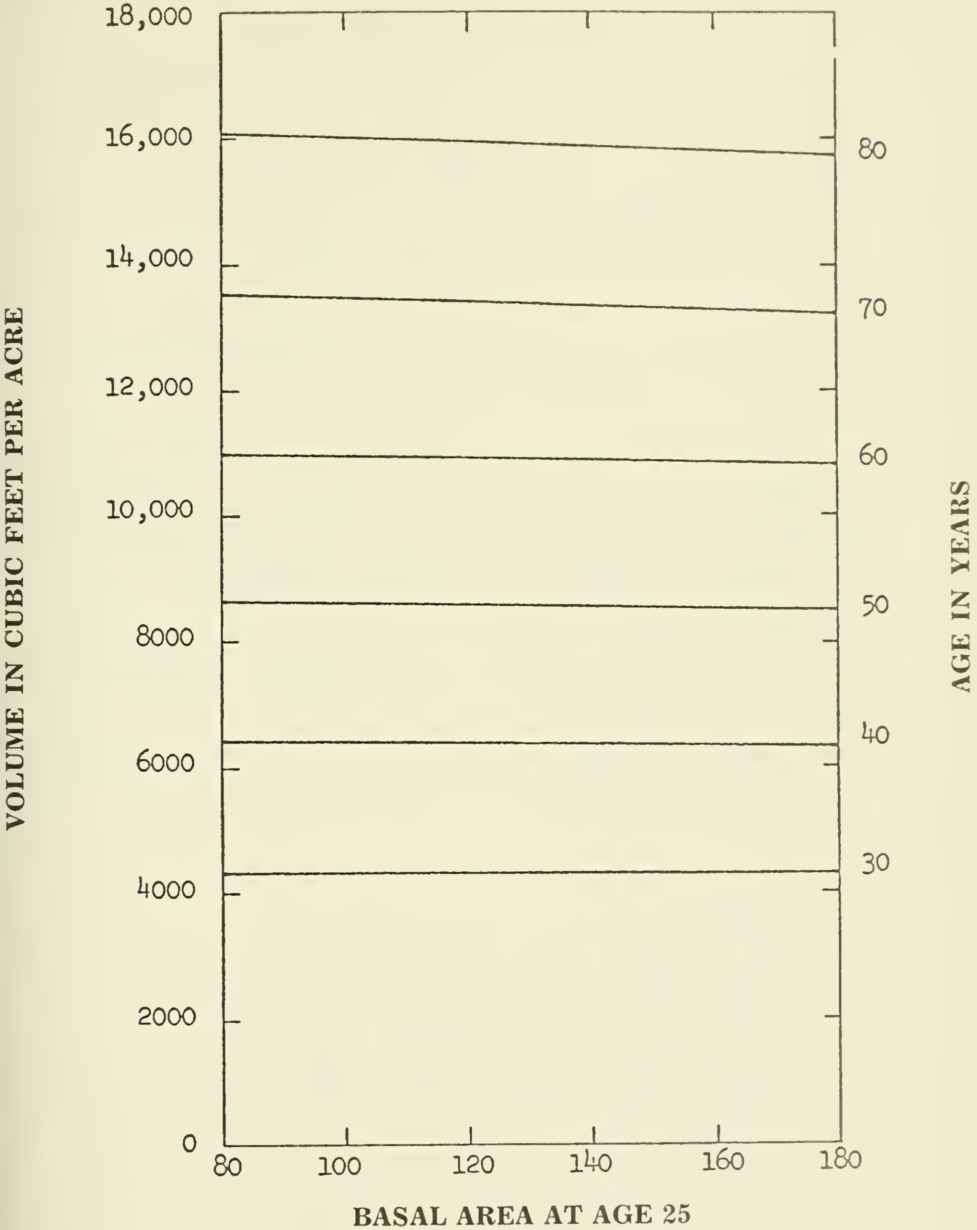


Fig. 6
Projected Yield in Cubic Feet per Acre
Initial Volume 3400 Cubic Feet

SUMMARY AND CONCLUSIONS

The purpose of this study was to find compatible growth and yield equations for yellow-poplar. A number of existing yield models were fitted to field data from 123 sample plots and were tested for best "fit." The equation that accounted for the largest percentage of the variation about the mean yield was selected for use in this study. Using this yield model, a tentative cubic-foot growth model was obtained by differentiation. To find an expression for basal area growth, a model for basal area was differentiated. Both the basal area and cubic-foot growth models were fitted to the data and regression coefficients solved for by least squares techniques.

Basal area and cubic-foot projection formulas were then obtained by integration of the basal area and cubic-foot growth equations. Using these projection formulas, total cubic-foot yield may be solved for any initial volume, age, or basal area.

This study resulted in the following equations:

Cubic-foot yield;

$$\text{Ln}V = 4.7123 + 0.0071(S) + 0.6167(\text{Ln}B) - 7.7335(A^{-1}) \qquad (2)$$

Basal area growth rate;

$$dB/dA = B(5.9358 - \text{Ln}B)A^{-1} \qquad (8)$$

Cubic-foot growth rate;

$$dV/dA = V(1.5261 - 0.0380\ \text{Ln}B)A^{-1} \qquad (9)$$

Projected basal area;

$$\text{Ln}B_P = 5.9358 - A_oA_P^{-1}(5.9358 - \text{Ln}B_o) \qquad (10)$$

Projected cubic-foot volume;

$$\text{Ln}V_P = \text{Ln}V_o + 1.3003(\text{Ln}\overset{A}{\blacksquare}_P - \text{Ln}\overset{A}{\blacksquare}_o) + A_o(A_P^{-1} - A_o^{-1})(0.0380\ \text{Ln}B_o - 0.2258) \qquad (12)$$

where:

LnV = the natural logarithm of inside bark cubic-foot plot volume,
S = site index in feet,
B = stand basal area in square feet,
A = stand age in years,
$dB\ dA$ = instantaneous rate of basal area growth,
$dV\ dA$ = instantaneous rate of cubic-foot growth,
B_0 = initial basal area,
A_0 = initial age,
B_P = projected basal area,
AA_P = predicted age,
V_P = projected cubic-foot yield per acre, and
V_0 = initial cubic-foot yield.

Although these equations are mathematically dependent, it should be noted that only one of the five significant equations was tested (Appendix A and B). Any of the others might have improved the compatible efficiency by improving the growth model. Then, too, no attempt was made to consider any of the other possible relationships between the dependent variables and the independent variables of age, site, and density. This study considered only normal forest and not such things as maximum and minimum densities, and various management regimes. These will be investigated in the future from the results of thinnings made in the established plots.

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APPENDIX A

Selection of a Yield Model

The following models were selected from the literature for use in this study. All were fitted to the data and tested by analysis of variance. As can be seen, any one of them could have been used, but only the one with the largest R^2 value was considered (Appendix B).

Yield Model	R^2	Standard Error	Citation
$\mathrm{Ln}V = b_0^{**} + b_1(S)^{**} + b_2(\mathrm{Ln}B)^{**} + b_3(A^{-1})^{**}$	99.9838	0.0095[1]	(a)
$\mathrm{Ln}V = b_0^{**} + b_1(A^{-1})^{**} + b_2(S)^{**} + b_3(SA^{-1})^{ns}$	99.9617	0.0146[1]	(b)
$\mathrm{Ln}V = b_0^{**} + b_1(S)^{**} + b_2(B)^{**} + b_3(A^{-1})^{**} + b_4(SA^{-1})^{ns} + b_5(BA^{-1})^{ns}$	99.9775	0.0134[1]	(c)
$V = b_0^{**} + b_1(B)^{**} + b_2(H)^{**} + b_3(BH)^{**}$	99.6563	18.4238	(d)
$V = b_0^{**} + b_1(S)^{**} + b_2(A)^{**} + b_3(H)^{**} + b_4(D)^{*} + b_5(B)^{**} + b_6(HB/100)^{**}$	98.7959	34.4842	(e)

* Significant at the 5 per cent level.

** Significant at the 1 per cent level.

ns Not significant at the 5 per cent level.

[1] Logarithmic scale.

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1963. Compatible growth and yield models for loblolly pine. Forest Sci. 9(3):354-371.

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APPENDIX B

Development of the Primary Equations

The following procedure is used to obtain compatible growth and yield equations. This is the technique used by Clutter (1963) in loblolly pine and is reproduced, in part, here.

Volume yield — From the Appendix A, the most significant yield model is:

$$\mathrm{Ln(Vol.)} = b_0 + b_1\mathrm{(Site)} + b_2\mathrm{(Ln\ Basal\ Area)} + b_3(A^{-1}) \qquad (1)$$

Following the regression, the equation became:

$$\mathrm{LnV} = 4.7123 + 0.0071(S) + 0.6167(\mathrm{LnB}) - 7.7335(A^{-1}) \qquad (2)$$

Noting that basal area is a function of age, equation (1) may be differentiated into:

$$dV/dA = b_2VB^{-1}(dB/dA) - b_3V(A^{-2}) \qquad (3)$$

where:

dV/dA = instantaneous growth in cubic feet, and
dB/dA = instantaneous growth in basal area.

Basal area growth — Although Equation 3 will provide for the cubic foot growth rate, the term dB/dA, for instantaneous growth in basal area, is not readily available. Therefore, the following model was used to define basal area:

$$\mathrm{LnB} = c_0 + c_1S + c_2A^{-1} + c_3(\mathrm{LnD})A^{-1} + c_4SA^{-1}$$

or,

$$\mathrm{LnB} - c_0 - c_1S = c_2A^{-1} + c_3(\mathrm{LnD})A^{-1} + c_4SA^{-1} \qquad (4)$$

where the symbols are the same as in Equation (1) and D is arbitrarily the beginning basal area at age 25. Differentiating Equation (4) with respect to age gives:

$$B^{-1}(dB/dA) = -A^{-1}(c_2A^{-1} + c_3(\mathrm{LnD})A^{-1} + c_4SA^{-1}) \qquad (5)$$

Substituting Equation (4) into Equation (5) and rewriting produces:

$$dB/dA = -B(\mathrm{LnB})A^{-1} + c_0BA^{-1} + c_1BSA^{-1} \qquad (6)$$

Substituting the right side of this equation into Equation (3) gives the cubic-foot growth expression:

$$dV/dA = -b_2V(\mathrm{LnB})A^{-1} + b_2c_0VA^{-1} + b_2c_1VSA^{-1} - b_3VA^{-2} \qquad (7)$$

Equation (6) was fitted to the basal area growth data with $(dB/dA + B(LnB)A^{-1})$ as the dependent variable. The second independent term BSA^{-1}, did not contribute significantly to the regression according to analysis of variance so the quantitative expression is:

$$dB/dA = BA^{-1}(5.9358 - LnB) \tag{8}$$

This accounts for 99.61 per cent of the variation about the mean basal area growth per acre, with a standard error of 0.12 square feet.

Volume growth — With the development of the basal area growth model, it is now possible to develop the expression for cubic foot growth. Equation (7) was fitted to the plot growth data. The variables VSA^{-1} and VA^{-1} were non-significant and removed from the model. After analysis this is:

$$dV/aA = VA^{-1}(1.5261 - 0.0380\ LnB) \tag{9}$$

and accounts for 93.81 per cent of the variation about the mean cubic foot growth per acre, with a standard error of 2.92.

Development of the Predicting Equations

Basal area growth — Equation (8) is the quantitative expression for instantaneous growth of basal area. To change this to a prediction equation, it is first solved for B:

$$B^{-1}dB = (5.9358 - LnB)A^{-1}dA$$

then integrate on the left from an initial basal area (B_o) to a predicted basal area (B_P) and on the right from initial age (A_o) to predicted age (A_P).

$$-Ln(5.9358 - LnB_P) + Ln(5.9358 - LnB_o) = LnA_P - LnA_o$$

and taking the antilog:

$$LnB_P = 5.9358 - A_oA_P^{-1}(5.9358 - LnB_o) \tag{10}$$

Volume growth — Solving Equation (9) for V gives:

$$V^{-1}dV = (1.5261 - 0.0380\ LnB)A^{-1}dA \tag{11}$$

Substituting Equation (10) into Equation (11) (using the notation B and A for B_P and A_P) produces:

$$V^{-1}dV = 1.3003\ A^{-1}dA + 0.2258\ A_oA^{-2}dA - 0.0380\ A_oA^{-2}(LnB_o)dA$$

Integrating on the left from initial volume (V_0) to predicted volume (V_P) and on right from A_0 to A_P (initial to predicted age) results in the following prediction equation for volume:

$$\begin{aligned} \operatorname{Ln}V_P = \operatorname{Ln}V_0 &+ 1.3003(\operatorname{Ln}A_P - \operatorname{Ln}A_0) \\ &+ A_0(A_P^{-1} - A_0^{-1})\ (0.0380\ \operatorname{Ln}B_0 - 0.2258) \quad (12) \end{aligned}$$